OCR
Oxford Cambridge and RSA

Monday 13 May 2024 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The quadratic equation $x^2 + ax + b = 0$, where a and b are real constants, has a root $2 - 3\mathrm{i}$.

(a) Write down the other root. **[1]**

(b) Hence or otherwise determine the values of a and b. **[3]**

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & a \\ -1 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$, where a is a constant.

(a) By multiplying out the matrices on both sides of the equation, verify that $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$. **[4]**

(b) State the property of matrix multiplication illustrated by this result. **[1]**

3 **(a)** Using standard summation formulae, write down an expression in terms of n for $\sum_{r=1}^{2n} r^3$. **[1]**

(b) Hence show that $\sum_{r=n+1}^{2n} r^3 = \frac{1}{4}n^2(an+b)(cn+d)$, where a, b, c and d are integers to be determined. **[5]**

4 **In this question you must show detailed reasoning.**

The roots of the cubic equation $x^3 - 3x^2 + 19x - 17 = 0$ are α, β and γ.

(a) Find a cubic equation with integer coefficients whose roots are $\frac{1}{2}(\alpha - 1)$, $\frac{1}{2}(\beta - 1)$ and $\frac{1}{2}(\gamma - 1)$. **[4]**

(b) Hence or otherwise solve the equation $x^3 - 3x^2 + 19x - 17 = 0$. **[3]**

5 **(a)** Find the volume scale factor of the transformation with associated matrix $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -1 & 0 & 2 \end{pmatrix}$. **[2]**

(b) The transformations S and T of the plane have associated 2×2 matrices $\mathbf{P}$ and $\mathbf{Q}$ respectively.

(i) Write down an expression for the associated matrix of the combined transformation S followed by T. **[1]**

The determinant of $\mathbf{P}$ is 3 and $\mathbf{Q} = \begin{pmatrix} k & 3 \\ -1 & 2 \end{pmatrix}$, where k is a constant.

(ii) Given that this combined transformation preserves both orientation and area, determine the value of k. **[3]**

6 You are given that $\mathbf{M} = \begin{pmatrix} 4 & -9 \\ 1 & -2 \end{pmatrix}$.

(a) Prove that $\mathbf{M}^n = \begin{pmatrix} 1+3n & -9n \\ n & 1-3n \end{pmatrix}$ for all positive integers n. **[6]**

(b) A student thinks that this formula, when $n = 0$ and $n = -1$, gives the identity matrix and the inverse matrix $\mathbf{M}^{-1}$ respectively.

Determine whether the student is correct. **[3]**

7 Three planes have equations

$$x + 2y - 3z = 0,$$
$$-x + 3y - 2z = 0,$$
$$x - 2y + kz = k,$$

where k is a constant.

(a) For the case $k = 0$, the origin lies on all three planes.

Use a determinant to explain whether there are any other points that lie on all three planes in this case. **[2]**

(b) You are now given that $k = 1$.

(i) Show that there are no points that lie on all three planes. **[3]**

(ii) Describe the geometrical arrangement of the three planes. **[1]**

8 In an Argand diagram, the point P representing the complex number w lies on the locus defined by $\left\{z : \arg(z-7) = \frac{3}{4}\pi\right\}$. You are given that $\text{Re}(w) = 1$.

(a) Find w. **[2]**

The point P also lies on the locus defined by $\{z : |z+3-9\text{i}| = k\}$, where k is a constant.

(b) Find the complex number represented by the other point of intersection of the loci defined by $\{z : |z+3-9\text{i}| = k\}$ and $\left\{z : \arg(z-7) = \frac{3}{4}\pi\right\}$. **[7]**

9 **In this question you must show detailed reasoning.**

Find a vector **v** which has the following properties.

- It is a unit vector.
- It is parallel to the plane $2x+2y+z=10$.
- It makes an angle of 45° with the normal to the plane $x+z=5$. **[8]**

END OF QUESTION PAPER